CHARACTERIZATION FOR GRAPHS WITHOUT INTERIOR VERTICES HAVING RECTANGULAR DUALS

A THESIS

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MANADO

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Presented as a Partial Fulfillment of the Requirements
in obtaining *Sarjana Teknik* Degree in Informatics Engineering Department

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Manado, 16 September 2004

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Ir. Noldi Watuna, MM. Ir. Simon Patabang, MT.

ABSTRACT

Lanny Sitanayah. (2004). *Characterization for Graphs Without Interior Vertices Having Rectangular Duals*.

VLSI physical design is one of the most popular applications of graph theory, in which rectangular drawing is used for VLSI floorplanning. *Floorplanning* is the process of placement flexible blocks, where circuits or functional modules can be represented by vertices of the graph and the adjacencies between modules can be represented by edges. The floorplan can be obtained by converting this graph into its rectangular dual.

A *plane graph* is a planar graph that is drawn/embedded in the plane with no two edges cross. A *rectangular drawing* of a plane graph G is a drawing of G in which each vertex is drawn as a point, each edge is drawn as a horizontal or vertical line segment without edge crossings, and each face is drawn as a rectangle. A *rectangular dual* of an n-vertex graph, $G = (V, E)$, is comprised of n non-overlapping rectangles with the following properties: (a) Each vertex $i \in V$, corresponds to a distinct rectangle i in the rectangular dual; (b) If (i, j) is an edge in E, then rectangles i and j are adjacent in the rectangular dual. Rectangular dual deals with rectangles, which in fact consist of vertices and horizontal-vertical edges, so basically rectangular dual is rectangular drawing.

Some algorithms to find the rectangular dual of a plane graph have been proposed by using *Proper Triangular Planar* (*PTP*) *graph*. A PTP graph is a connected planar graph that satisfies the following properties: (a) Every face (except the exterior) is a triangle (i.e., bounded by three edges); (b) All internal vertices have degree ≥ 4; (c) All cycles that are not faces have length ≥ 4. The PTP graph has rectangular dual.

In this thesis we establish a necessary and sufficient condition as characteristics for a plane graph without interior vertices having rectangular dual. As a result, we obtain *Proper Quadtri Exterior Plane* (*PQEP*) *graph*. The PQEP graph G is a plane graph without interior vertices, which all vertices of G have degree > 1 and satisfies the following three conditions: (a) G has no more than four exterior vertices of degree 2; (b) All interior faces of G are triangles or quadrangles; (c) Interior quadrangle face of G does not contain cut-vertex. PQEP graphs have rectangular duals.

Keywords: Graph, Plane Graph, Rectangular Dual, Proper Quadtri Exterior Plane, Floorplanning.

ABSTRAK

Lanny Sitanayah. (2004). *Characterization for Graphs Without Interior Vertices Having Rectangular Duals*.

VLSI physical design adalah salah satu aplikasi *graph theory* yang sangat terkenal, dimana *rectangular drawing* digunakan untuk *VLSI floorplanning*. *Floorplanning* adalah proses penempatan blok-blok yang fleksibel, dimana sirkuit atau modul fungsional dapat direpresentasikan oleh *vertex* dari *graph* dan kedekatan di antara modul-modul dapat direpresentasikan oleh *edge*. *Floorplan* dapat diperoleh dengan mengubah *graph* menjadi *rectangular dual*-nya.

Plane graph adalah *planar graph* yang digambarkan pada bidang datar tanpa perpotongan *edge*. *Rectangular drawing* dari *plane graph G* adalah gambar dari *G* dimana setiap *vertex* digambarkan sebagai titik, setiap *edge* digambarkan sebagai garis horisontal atau vertikal tanpa perpotongan, dan setiap *face* digambarkan sebagai segi empat. *Rectangular dual* dari *graph* dengan *n-vertex*, $G = (V, E)$, terdiri dari *n* segi empat yang tidak tumpang tindih dengan sifat: (a) Setiap *vertex* $i \in V$, berhubungan dengan sebuah segi empat berbeda *i* di *rectangular dual*; (b) Jika (i, j) adalah sebuah *edge* di *E*, maka segi empat *i* dan *j* berdekatan di *rectangular dual*. *Rectangular dual* berhubungan dengan segi empat, yang terdiri dari *vertex-vertex* dan *edge-edge* horisontal-vertikal, sehingga pada dasarnya *rectangular dual* adalah *rectangular drawing*.

Beberapa algoritma untuk mencari *rectangular dual* dari *plane graph* telah dikemukakan dengan menggunakan *Proper Triangular Planar* (*PTP*) *graph*. PTP *graph* adalah *connected planar graph* yang memenuhi sifat: (a) Setiap *face* (kecuali eksterior) adalah segi tiga (yaitu dibatasi oleh tiga *edge*); (b) Semua *vertex* internal memiliki *degree* ≥ 4; (c) Semua *cycle* yang bukan *face* memiliki *length* ≥ 4. PTP *graph* memiliki *rectangular dual*.

Pada skripsi ini, dibuktikan kondisi *necessary* dan *sufficient* sebagai karakteristik sebuah *plane graph* tanpa *vertex* interior yang memiliki *rectangular dual*. Hasilnya, diperoleh *Proper Quadtri Exterior Plane* (*PQEP*) *graph*. PQEP *graph G* adalah sebuah *plane graph* tanpa *vertex* interior, dimana semua *vertex G* memiliki *degree* > 1 dan memenuhi kondisi: (a) *G* tidak memiliki lebih dari empat *vertex* eksterior dengan *degree* 2; (b) Semua *face* interior *G* adalah segi tiga atau segi empat; (c) *Face* segi empat interior *G* tidak mengandung *cut-vertex*. PQEP *graph* memiliki *rectangular dual*.

Kata kunci: *Graph, Plane Graph, Rectangular Dual, Proper Quadtri Exterior Plane, Floorplanning*.

ACKNOWLEDGEMENTS

Praise be the Lord because of His blessing and guidance, this thesis has finished already.

First of all, I would like to thank my supervisor Dr. Benny Pinontoan, M.Sc. for introducing me this beautiful and fascinating graph theory, the valuable references, and for this interesting topic of research. I really appreciate his precious time given in supervising, valuable advice, help, continual encouragement, extraordinary patience in reading my drafts and making corrections, without which this thesis would have not been possible.

I would like to express my special thanks to Dr. Xin He, professor of Department of Computer Science and Engineering, State University of New York at Buffalo, for the useful correspondence about the topic of this research and convincing me that the research would be possible.

I would like to thank the other examiners, Ir. Armein Z. R. Langi, M.Sc., Ph.D. and Dr. Ir. Rajesri Govindaraju, M.Sc., lecturers from Institut Teknologi Bandung, for the valuable advice, discussions, suggestions and corrections when this thesis was defended on 16 September 2004.

I would like to thank my English reviewer, Fanny Novita A., SPd. for the time spent in writing corrections, valuable discussions and suggestions throughout the revision.

Also, I would like to thank my lecturer in Digital System subject, Lianly Rompis, ST. for the advanced understanding in Digital Circuit and example needed in this thesis.

Special thanks goes to Dr. Johanis J. Montolalu, Pr, Rector of De La Salle Catholic University Manado; Ir. Noldi Watuna, MM., Dean of Engineering Faculty; and Ir. Simon Patabang MT., Head of Informatics Engineering Department; who have shared in much of my graduate life and provided continual emotional support and understanding.

I would like to acknowledge with sincere thanks the references provided by Chandra Paringkoan, Indra Irawan Isa, Lucia Margareth, Senly Sambuaga and Ruland Rantung.

Finally, I would like to thank anybody who contributes much in my life, especially during this research. Also, for the person who makes me write the theoretical background twice. You know who you are.

"***There will be no ending without a start***"

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$=$	equal	11
$\neq$	not equal	15
$>$	greater than	30
$\geq$	greater than or equal	1
$\leq$	less than or equal	1
$\wedge$	and	30
$\vee$	or	30
$\neg$	not	30
$\Leftrightarrow$	equivalent	30
$\Rightarrow$	implies	30
$\{\ldots\}$	set	12
$\|X\|$	cardinality of set X	21
$\in$	element of a set	13
$\notin$	not element of a set	15
Δ	maximum degree of a graph	13
$deg(f)$	degree of a face f	20
$deg(v)$	degree (or valence) of a vertex v	13
E	set of edge(s)	12
$E*$	set of dual edge(s)	23
F	set of face(s)	20
$F*$	set of dual face(s)	23
$G - S$	subgraph containing the vertices of a graph G not in a proper subset S of vertices of G, and the edges of G not incident on each vertex in S	15
$K_{m,n}$	complete bipartite graph that has m vertices in one of its bipartition subsets and n vertices in the other	14
K_n	complete graph on n vertices	13
$k(G)$	number of component of a graph G	18
$k_e(G)$	edge connectivity of a connected graph G	19
$k_v(G)$	vertex connectivity of a connected graph G	19
$N(v)$	neighborhood of a vertex v	13
V	set of vertex (vertices)	12
$V*$	set of dual vertex (vertices)	23